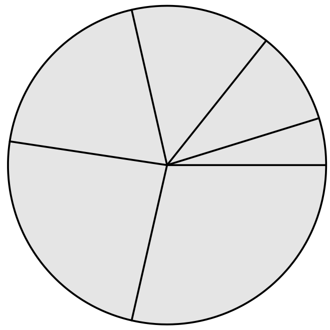
Maclaurin Olympiad 2018

M1. The sum of the squares of two real numbers is equal to fifteen times their sum. The difference of the squares of the same two numbers is equal to three times their difference.

Find all possible pairs of numbers that satisfy the above criteria.

M2. The diagram shows a circle that has been divided into six sectors of different sizes.



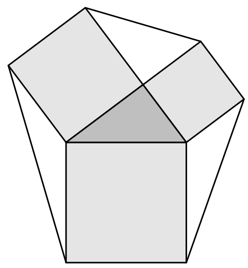
Two of the sectors are to be painted red, two of them are to be painted blue, and two of them are to be painted yellow. Any two sectors which share an edge are to be painted in different colours.

In how many ways can the circle be painted?

M3. Three positive integers have sum 25 and product 360.

Find all possible triples of these integers.

M4. The squares on each side of a right-angled scalene triangle are constructed and three further line segments drawn from the corners of the squares to create a hexagon, as shown. The squares on these three further line segments are then constructed (outside the hexagon).



The combined area of the two equal-sized squares is 2018 cm^2.

What is the total area of the six squares?

M5. For which integers n is $\dfrac{16(n^2 - n - 1)^2}{2n - 1}$ also an integer?

M6. The diagram shows a triangle ABC and points T, U on the edge AB, points P, Q on BC, and R, S on CA, where:

(i) SP and AB are parallel, UR and BC are parallel, and QT and CA are parallel;

(ii) SP, UR and QT all pass through a point Y; and

(iii) $PQ = RS = TU$.

Prove that

$$\frac{1}{PQ} = \frac{1}{AB} + \frac{1}{BC} + \frac{1}{CA}.$$